Quasi-Chaotic Digital behaviour in an Optically-Processing Element

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ABSTRACT

Digital chaotic behaviour in an Optically-Processing Element is reported. It is obtained as the result of processing two fixed train of bits. The process is performed with an Optically Programmable Logic Gate. Possible outputs, for some specific conditions of the circuit, are given. These outputs have some fractal characteristics, when input variations are considered. Digital chaotic behaviour is obtained, by using a feedback configuration. A random-like bit generator is presented.

1.- INTRODUCTION

The dynamical behaviour of nonlinear systems have been studied intensively in the last years. Most of the work concerning the temporal operation of such systems has been done with analogical signals and on the assumption that nonlinear optical systems are described by differential equations. Indeed, this mathematical model is a good approximation of all systems with time constants large compared to the mean group delay or roundtrip time of the feedback loop. Taking into account, however, a finite feedback delay comparable to or greater than the combined time constants of all system components, the dynamics of the system is given by difference-differential equations. Ikeda[1] was the first to apply this type of analysis to a ring cavity system with a nonlinear medium. From his results, he concluded that new types of instabilities should be found in such systems yielding periodic and even chaotic solutions. Furthermore, Okada and Takizawa[2] investigated the effect of a delayed feedback in a hybrid electrooptic system with the restriction that the delay is less than or comparable to the response time of such a system. Neyer and Voges[3], finally, studied the pure effect of the feedback delay on the behaviour of a electrooptic system, neglecting all time constants of the system components.

The optical structure behaviour which is going to be reported here is a basic configuration, reported previously by us[4], able to process two input binary signals, being the output two logical functions. The type of processing is related to the eight main Boolean Functions: AND, OR, XOR, NAND, NOR, XNOR, ON & OFF. Programmable ability of the two outputs, as it will be described in this paper, allows the generation of several data-coding for optical transmission.

The main objective of this paper is to demonstrate the possibility to apply the above mentioned structure to the generation of periodic and even chaotic solutions. A precise analyze of the output characteristic, versus the main variable parameters (as control signal level, data signal level, ...), has given some results which can be described with the fractal and chaotic theories. These theories allow to understand the reported behaviour .

2.- OPTICAL CONFIGURATION OF THE OPTICALLY-PROGRAMMABLE DIGITAL CIRCUIT

The present optically-programmable digital circuit has been already reported[4] as a Programmable Logic Gate. A brief description on its method of operation, as well as the way it has been implemented, is going to be summarized here. We are going to do that because we think it is necessary to clarify some points needed for its present application. A major discussion, about the different possible devices to be employed and the transmission medium used for its implementation, can be found on reference[5].

A block diagram of the circuit is show in Fig. 1. As it can be seen, the circuit is composed by two optical devices, P and Q, with a non-linear behaviour. The output of each one of them corresponds to the two final outputs, O_1 y O_2, of the cell. The possible inputs to the circuit are four. Two of them are for the input data, I_1 e I_2, and the other two, *g* and *h*, for control signals. The way these four inputs are distributed inside the circuit it is also represented in Figure 1. The corresponding inputs to the non-linear devices are based on these signals plus some others coming from inside the own cell.

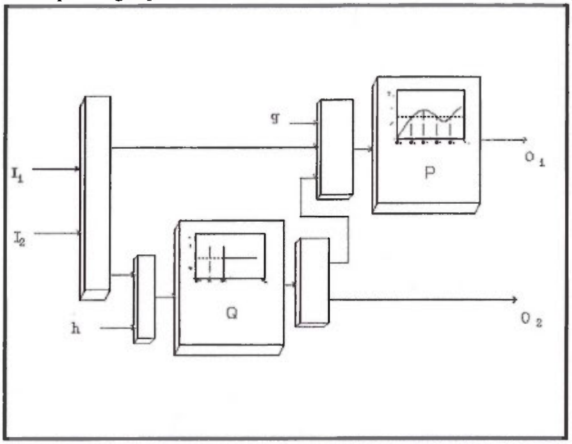

Figure 1.- Block diagram of the optical-programmable logic circuit.

The practical implementation we have carried out of the processing element has been based on an optolectronical configuration. Lines in Fig. 1 represent optical multimode fibers. The indicated blocks, placed in order to combine the corresponding signals, are conventional optical couplers. In this way, the inputs arriving to the individually devices are multilevel signals. The working levels are:

a) for device P:

$$I_P=(I_1+I_2)/2+g+O_2/2 \tag{1}$$

and

b)for device Q:

$$I_Q=(I_1+I_2)/2+h \tag{2}$$

Therefore, the output of the P-device depends on the control signal *g*, plus one half of the output, O_2, of the Q-device. The output of the Q-device depends only on the control signal *h*. We understand as "output" the type of processing, or logical function, which executes each one of the devices, on the two binary data inputs.

The characteristics of the non-linear devices are also shown in Fig.1. Device Q, corresponds to a thresholding or switching device, and device P is a multistated device, being the ideal response of our non-linear optical device the one represented in Fig.1. Because the input signal is a multilevel signal (equations 1-2), the output depends on the relations between:

a) level of a bit "1",
b) level of the control signal and
c) the level, intrinsic to the device, in which it switches from one state to another.

In order to clarify the above facts, in fig.2 we have represented how is obtained one of the possible outputs for each device. The previous mentioned parameters are indicated in the same figure. These devices has been carried out, experimentally, with an optoelectronic approximation.

However, in the results to be reported here, we have not paid too much attention to the experimental implementation but to its computer simulation. It has been done with the MATLAB™ program and the SIMULAB™ application.

On the behaviour simulation of the optical-programmable logic circuit a normalization of one bit "1" at the input of the cell, I_1 or I_2, of value 1 has been considered. As variable parameters we have taken:

i) the decision levels of each device, d_Q and d_P (see fig.2), and,
ii) both controls signals, *g* and *h*.

Because the P-device is more complex than the Q-, the approximation that has been used is the one represented in fig.2. We have considered five levels of decision. They are equidistant with respect to the first one d_{P1}. However, there are just three input levels where the output is able to switch. This fact is due to the real characteristics of the optical non-linear

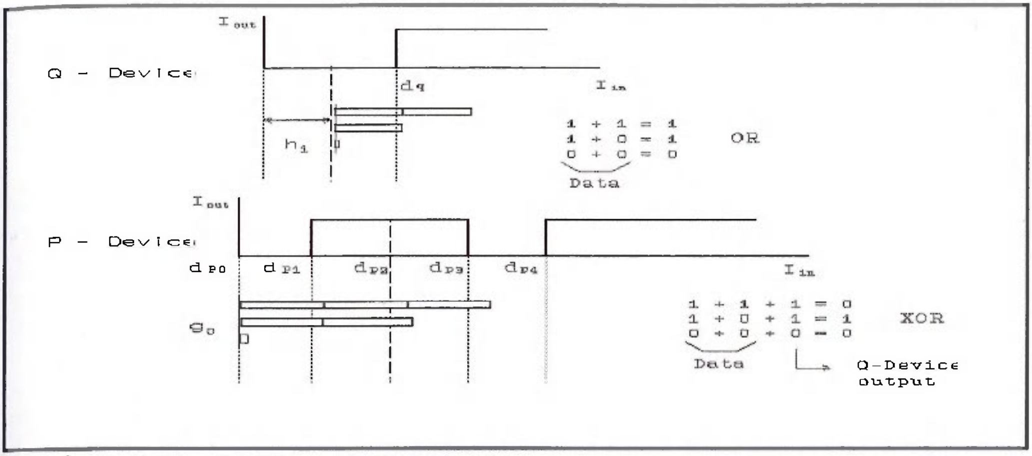

Figure 2.- Example of Input/Output characteristic for Q-device and P-device.

device considered.

In a practical situation, and as a first application of the circuit, the decision levels for each device have well defined values. In this case, and by fixing the decision level to $d_Q=1$ and $d_{P1}=0.5$, several functions can be obtained. The way to achieve this fact is by changing both control signal from 0 to 4. The functions that have being obtained are summarized on Table I.

TABLE I.- Output functions of the optical-programmable logic circuit.

Q - Control Signal --- / P - Control Signal	Q-Output: AND 0-0.4 / P - Output	Q-Output: OR 0.5-0.9 / P - Output	Q-Output: ON 1.0-2.0 / P - Output
0-0.4	XOR	XOR	NAND
0.5	NAND	NOR	NOR
0.6-0.9	ON	XNOR	XNOR
1.0	XNOR	XNOR	AND
1.1-1.4	XNOR	ON	OR
1.5	AND	OR	OR
1.6-2.0	OR	OR	ON
2.0-2.5	ON	ON	ON

Another application of this cell has the possibility of controlling the decision levels. These levels are intrinsic to each device. In this case, and by maintaining the same normalization as before, we have obtained some other different tables, equivalents to Table I. Each one of them corresponds to the various decision level values.

In order to be as brief as possible, from now on, we will analyze and report just the results corresponding to the first output, O_1. As it can be seen in Fig. 1, it is related to the more complex device P. A similar study can be done for the Q-device.

The behaviour of O_1-output, for one of the possible types of functions at output O_2, is shown in Figure 3. The axis represent, in ordinates, the power level of control signal h, and in x-axis the power level of decision signal, d_{P1}.

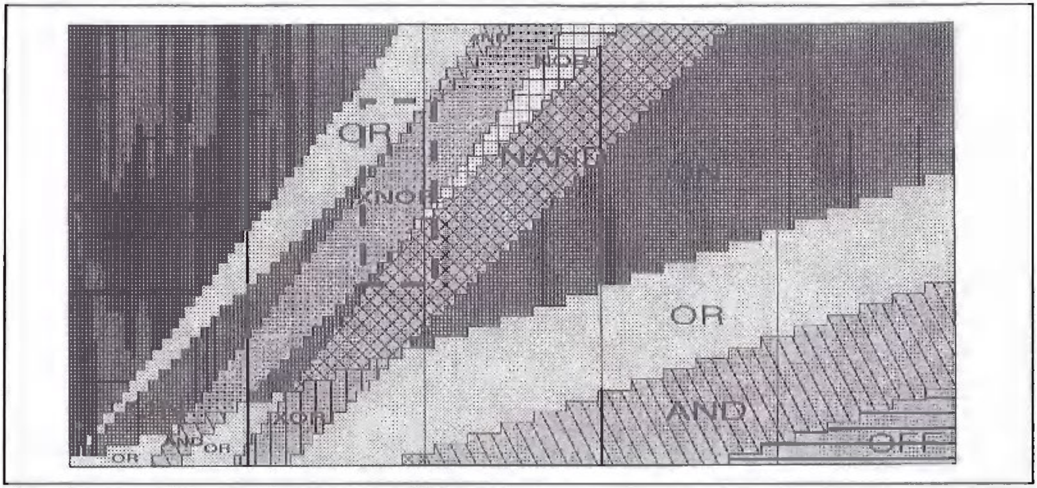

Figure 3.- O_1-output behaviour of the optical-programmable logic circuit.

As we can see, a large variety of logic functions are obtained in a very simple way. Moreover, a fact needs to be pointed out. It concerns to the extreme sensitivity of the output logic functions with respect to the control signals and to the decision levels. This fact will of a great importance for further considerations on the working conditions.

3.- SOME CONSIDERATIONS ABOUT THE FRACTAL CHARACTERISTICS OF THE OUTPUT.

If we analyze an specific area of Figure 3, as the one represent inside the dashed rectangle, we find a curious behaviour. The circuit characteristics depend on the decimal precision that we have taken for the input control signal and the decision level. This analysis have been performed by computer.

Figures 4.1-4.3 show the graphic evolution and dependence with the two indicated parameters, control signal and decision level, at the above region. As we can see, the obtained graph depends strongly on the adopted precision. In Fig. 4.1, control signal was moved by steps of 0.1 and decision level by steps of 0.01. In Fig. 4.2.a, these steps were 0.01 and 0.01; in Fig. 4.2.b, 0.1 and 0.001; and, finally, in Fig. 4.3, 0.01 and 0.001.

In order to get a closer look of the obtained results, we have studied their fractal dimensions. As we know from any book on this topic[3], several are the possibilities to obtain these dimensions. Because the Hausdorff measure and dimension are somehow complicated to be employed here we have adopted the more conventional polygonal approximation and the

Box-Counting method.

As it is known, the fractal dimension of any surface is given, in the case of the polygonal approximation, by an expression as

$$u = c\left(\frac{1}{s}\right)^d \tag{3}$$

where u is the obtained length and s is the compass setting that gives a measure of precision. d is the slope of the fitting line and the key to the fractal dimension of the underlying object. Another way to express the above equation is

$$d = \frac{\log u}{\log \frac{1}{s}} \tag{4}$$

With respect to the second way of calculating the fractal dimension, we have considered the number of "boxes" that intersect the borders of the taken area. In this case the corresponding dimension is given by

$$D_{k+1,k} = \frac{\log N(s^{-(k+1)}) - \log N(s^{-k})}{\log (s^{k+1}) - \log (s^{k})} \tag{5}$$

where s^k is the value of the adopted unit for the k-th measure step and $N(s^k)$ corresponds to the number of boxes counted in such a step.

We have applied, as we have indicated previously, the two above mentioned methods to three different results of the graphical representation of our output functions. Each one of them corresponds to different precision in the representation units. The results are summarized in Table II. As we can see, almost in every one of them the result is the same one: very close to 0.9.

Although this result is not a clear indication of an irregular behaviour in our system, it gives the indication of the importance of precision when some result is obtained. Some other considerations could be obtained from the above results, but they will be object of future works.

TABLE II.- Parameter values for fractal dimension.

	S	u	d	$D_{k+1,k}$
A	0.01/0.001	554	0.9145	D_{AB} = 0.98
B	0.01/0.01	58	0.8817	
C	0.1/0.01	7	0.8451	D_{AC} = 0.91

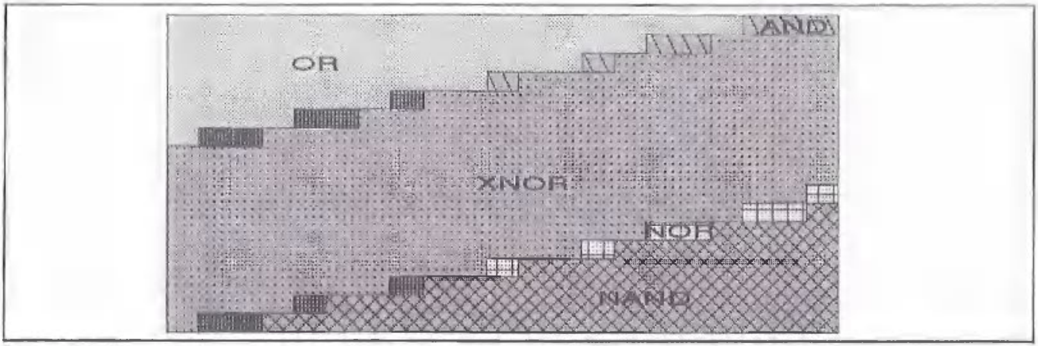

Figure 4.1.- Close view of dashed rectangle in Figure 3. Precision of control signal and decision level: 0.1, 0.01.

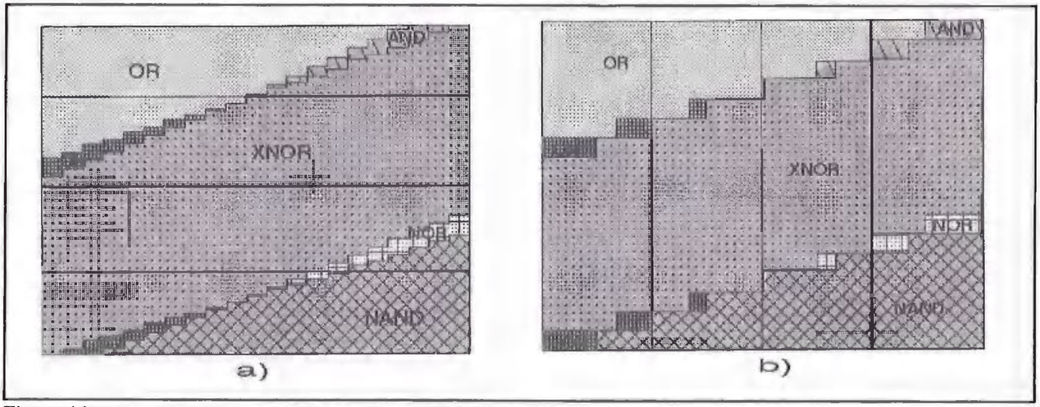

Figure 4.2.- Output functions with the precision of control signal and decision level: a) 0.01, 0.01; b) 0.1, 0.001.

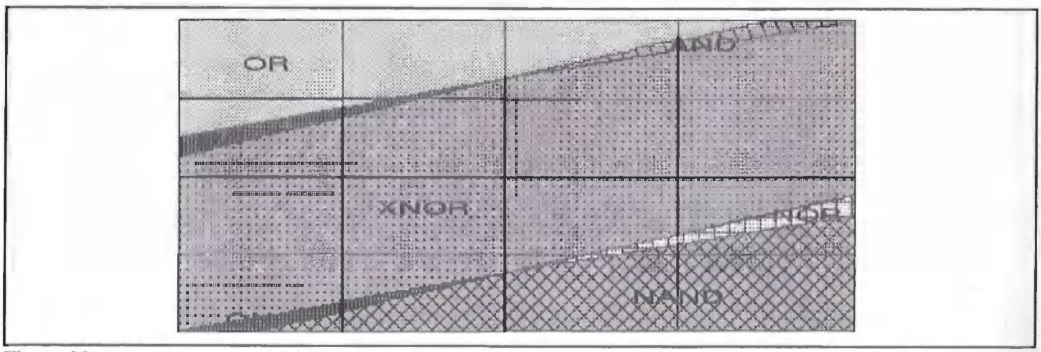

Figure 4.3.- Output functions with the precision of control signal and decision level: 0.01, 0.001.

4.- QUASI-CHAOTIC BEHAVIOUR WITH FEEDBACK

As it has been seen, our Optically-Programmable Circuit offers a fractal-like behaviour, for certain relations between parameters of the circuit. It is expected, in the same way, that a nonlinear behaviour could be obtained if some kind of feedback is applied. In references (2) and (3) some examples of this type of behaviour are reported.

In order to study the response of our circuit with feedback, some minor modifications are needed. The first one should be to introduce a feedback from one of the two possible outputs to one or both of system inputs. Moreover, according to previous studies in this field, the introduced feedback should have some delay. Because the results we are going to obtain will be the result of a computer simulation, another delay should be needed. It corresponds to the response time of the internally simulated nonlinear devices. In general, a periodic behaviour could be the normal output of the system. But, under some conditions, this is not always true. The output, as we will see, is not periodic with some parameters values of the system. We will show some of these situations.

When feedback is applied to the system, two are the possibilities. Because the P-device output has more possible different functions, depending on its control signal (see Table I), than the Q-, we have used this output for feedback. The output goes to the control signal input, *g*, of P-device. No other additional control signal is used. Figure 5 shows the final circuit with feedback.

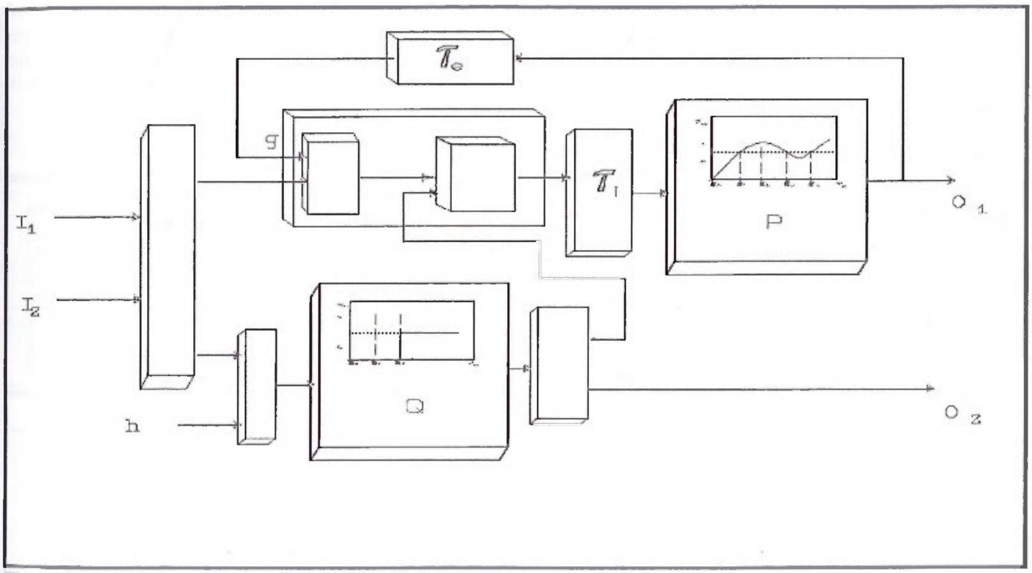

Figure 5.- Optical-Programable Logic Circuit with Feedback.

The first analysis that we have performed, by simulation tools, considered null delay times. This situation has not an algebraic solution and no data were obtained.

The circumstances are strongly different if we introduce finite delay times, namely, internal and external delays.

According to previous studies[2-3], the situation with more probability to give a periodic or even chaotic solution is when internal time delay is shorter than the external one. In any case, input has been a regular train of pulses. The real input to the device P, before the feedback takes place, is shown in Fig. 6. As it can be seen, it is a multilevel signal corresponding to the addition of the two inputs. The period of this signal corresponds to a time of 14 bits.

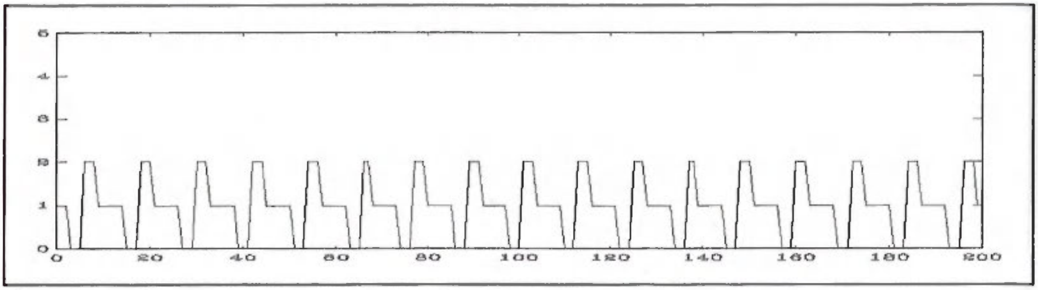

Figure 6.- Input data signals to the optical-programmable logic circuit.

If the ratio between internal delay time and external delay time is smaller than 1, we obtain a periodic situation. The period of this signal is strongly dependent on the ratio value. In the particular case, where external time delay is 200 and internal delay are 2, 4 and 12, obtained results are summarized in Figs. XXX XXX and XXX. Moreover, in order to get a rapid view of these results, they are indicated in Table III. An interesting result is the duplication in period time when the ratio between delays get smaller. It goes from 70 to 280. Hence, we have obtained frequency doubling. And this is one of the best indications of the route to chaos.

TABLE III.- Characteristics of the output signals, according to the delay times.

t_p	τ_e	τ_i	τ_i/τ_e	Period
14	200	2	0.01	280
14	200	4	0.02	140
14	200	12	0.06	70

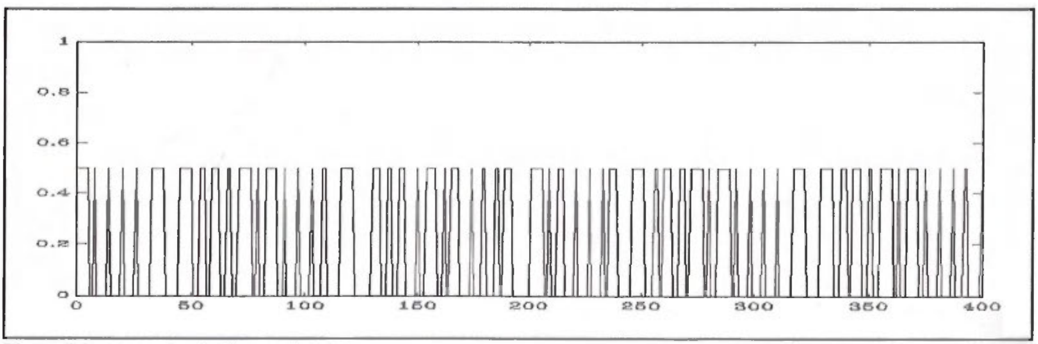

Figure 7.1.- Output signal with a period of 280.

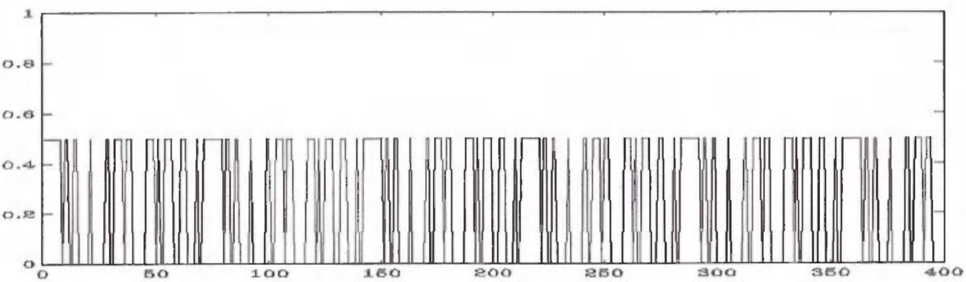

Figure 7.2.- Output signal with a period of 140.

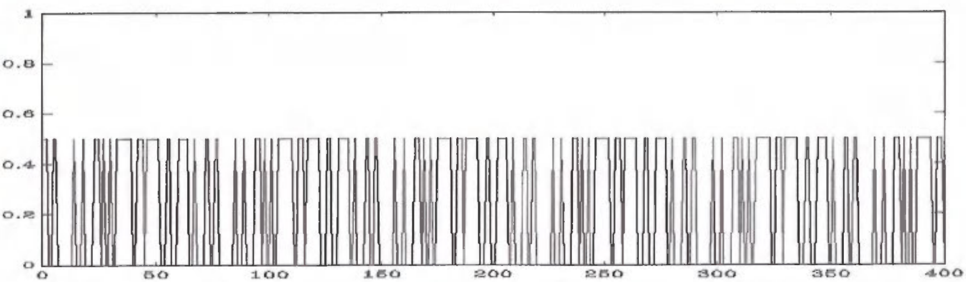

Figure 7.3.-Output signal with a period of 70.

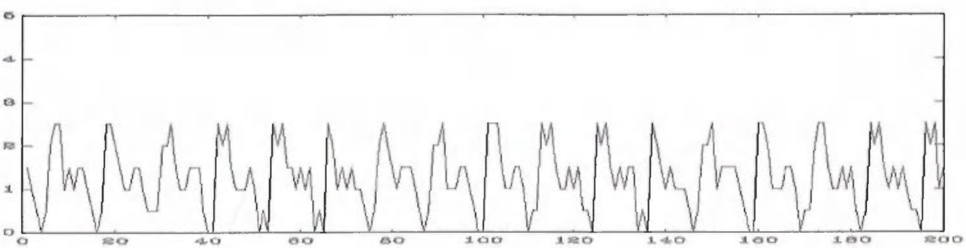

Figure 8.-Input signal with feedback to P-device.

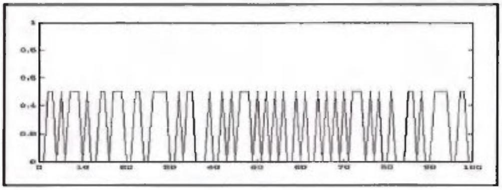

Figure 9.1

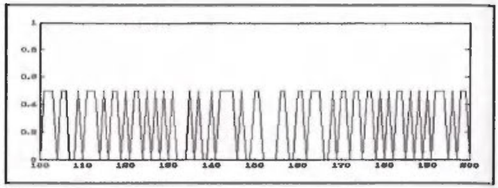

Figure 9.2

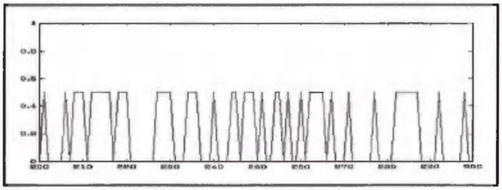

Figure 9.3

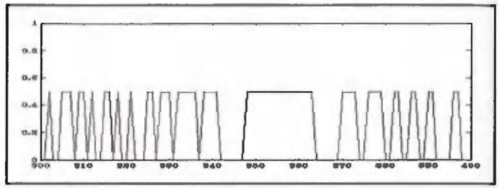

Figure 9.4

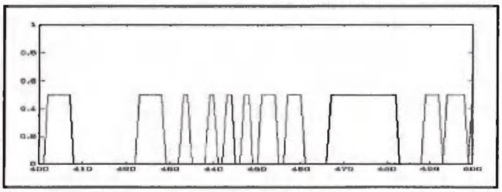

Figure 9.5

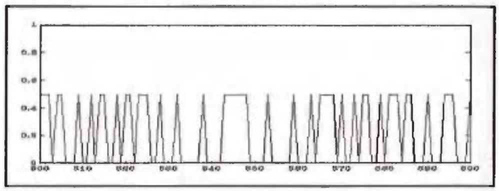

Figure 9.6

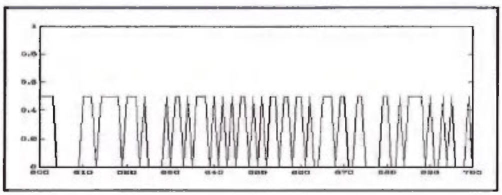

Figure 9.7

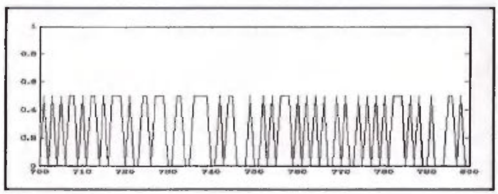

Figure 9.8

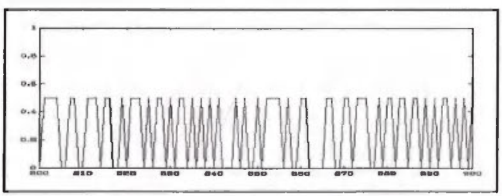

Figure 9.9

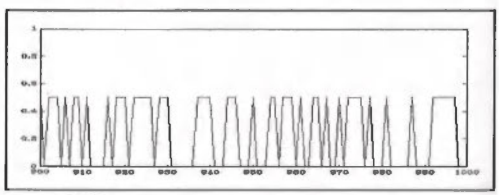

Figure 9.10

Figs 9.1 - 9.10.- Time evolution of output signal when internal delay time is zero.

A further point must be taken into account. Values giver at Table III do not correspond to the transition points between different periods. They are in a range where period remains constant. If we calculate the equivalent to the Feigenbaum ratio for the indicated values we obtain 4. But if higher order transition points were considered a number, closer to 4.669201..., should have been obtained.

According to the obtained results, we have made the internal response time equal to zero. Input signal, corresponding to feedback plus input data, to P-device control gate, is shown in Fig. 8. Output signals are given in Figs. 9.1-9.10. No indication of a possible periodic behaviour has been obtained for longer intervals of time. These results, because they have been achieved after a Period-Doubling Route, clearly indicate that a certain type of chaos is present.

5.- CONCLUSION

A new type of digital chaos has been obtained. The basic structure is an Optically Programmable Digital Cell, reported previously by us, as the main block for a possible optical computer. A feedback was applied and internal and external time delays considered. Although previous results with this Cell were experimental, this paper concerns with its computer simulation. Obtained results indicate that the same system is able to be employed as random optical bits generator. Moreover, because we have been able to know how is the route to this type of digital chaos, some more information about this phenomenon could be obtained from it. Further work on this topic will be reported elsewhere.

6.-ACKNOWLEDGEMENTS

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